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Flux backgrounds in 2D string theory

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Abstract: We study RR flux backgrounds in two dimensional type 0 string theories. In particular, we study the relation between the 0A matrix model and the extremal black hole in two dimensions. Using T-duality we find a dual flux background in type 0B theory and propose its matrix model description. When the Fermi level μ is set to zero this system remains weakly coupled and exhibits a larger symmetry related to the structure of flux vacua. Finally, we construct a two dimensional type-IIB background as an orbifold of the 0B background.

Keywords: Superstrings and Heterotic Strings, D-branes, Black Holes in String Theory, Matrix Models.

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1. Introduction and summary

Two dimensional string theory provides a simple and tractable system, where perturbative string dynamics can be studied exactly using the connection with the $c = 1$ matrix model [1] (for reviews see e.g. [2, 3, 4]). Motivated by the relation between the $c = 1$ matrix model and the dynamics of unstable D0-branes [5, 6, 7], recently [8, 9] proposed a non-perturbative matrix model description for type 0 string theory in two dimensions. According to [8, 9], the type 0B matrix model is defined as a double Fermi sea version of the usual $c = 1$ matrix model, whereas the 0A matrix model is defined as a theory of complex random matrices, which describe open string degrees of freedom in the $D0 - \overline{D0}$ system. These models do not suffer from non-perturbative instabilities encountered in the bosonic string theory [10, 11, 4], and lead to consistent unitary theories as shown by [12] by applying the results of [13] (see also e.g. [14]–[39] for recent discussions of $c = 1$ matrix model and two dimensional string theory).

In addition to non-perturbative stability, an advantage of these models compared to the bosonic string theory is that they admit backgrounds with RR-fluxes. In the 0B theory, such backgrounds can be obtained by considering asymmetric Fermi levels. On the other hand, in the 0A theory the RR flux is determined by the parameter q, which enters the definition of the dual matrix model and can be interpreted as the net D0-brane charge [9]. It was pointed out in [21, 12](see also [9]), that after projecting onto the singlet sector, the 0A model reduces to the Jevicki-Yoneya model [40], which was originally proposed as a candidate for describing the two dimensional black hole [41]. More recently, a different matrix model dual of the black hole was proposed in [42].

Motivated by these developments, in this paper we investigate 0A and 0B flux backgrounds from the viewpoint of both the matrix model and the two-dimensional space-time gravity. The 0A matrix model with non-zero value of the parameter q can be weakly coupled (and non-perturbatively stable) even when the Fermi level μ is zero. We argue that in the supergravity approximation, its ground state can be identified with the extremal black hole [43] found in the context of flux compactifications of type-II string theory down to two dimensions [44, 45]. In particular, we identify the energy of the 0A matrix model with the ADM mass of the extremal black hole and comment on the UV/IR relation in open/closed string duality.

It is also useful to consider T-dual backgrounds in type 0B theory with non zero RR flux. By analyzing the partition functions of the 0A and 0B matrix models we demonstrate their relation under T-duality. We propose a Fermi sea description of a type 0B background with $\mu = 0$ and large lightlike RR flux. It is curious to note, that it has similar perturbative structure with the usual 0B background with zero flux and non-zero tachyon condensate. Using the results of [46], we also construct a variety of flux solutions in the effective space-time gravity theory, including the solution T-dual to the extremal black hole and various time-dependent backgrounds with flux. We provide their Fermi sea description in the matrix model.

Interestingly, the 0B matrix model configuration we analyze in detail is invariant under a $\mathbb{Z}_2$ symmetry which interchanges the fermions and the holes. This symmetry is equivalent to GSO projection on the world-sheet. Thus orbifolding by this symmetry, gives a two-dimensional type-IIB string theory background. In conclusion we have a $c = 1$ matrix model description of two-dimensional type-IIB theory.

The paper is organized as follows. In section 2 we discuss the relation between the 0A matrix model and the extremal black hole solution of [43]. In section 3 we study the 0B theory with non-trivial large RR flux and propose a dual Fermi sea description. An SU(2) symmetry which mixes particles and holes enters the description in an interesting way. In section 4 we consider classical solutions in 0B space-time gravity. We find a static solution which describes the flux background as well as time dependent solutions which correspond to time dependent Fermi seas. In section 5 we describe a matrix model dual of a type-IIB background as an orbifold of the 0B matrix model.

Note added: while we were completing this manuscript, we noticed the paper [47], which has partial overlap with section 3. After submitting this paper we received [48] which also discusses the relation of the 0A model to the extremal black hole.

2. Flux condensation in 0A theory and black holes

2.1 The 0A matrix model

The 0A matrix model in two dimensions [9, 8] is defined as a sum over complex rectangular random matrices. The model is expected to describe the dynamics of $N + q$ D0-branes and

N anti D0-branes in the theory. More precisely, in type 0A theory we have two types of D0-branes, electric and magnetic. Each type of D0-branes couple to a RR 1-form potential.[1] In the presence of a Liouville potential in the worldsheet theory, or a tachyon background $T = \mu e^{\phi}$ in spacetime, only one type of brane is physical [49, 50, 8, 9]. The type of brane allowed depends on the sign of μ. For positive μ, we have electric branes while for negative μ magnetic branes.

The field content of the model consists of an $(N+q) \times N$ complex matrix Φ, corresponding to the open string tachyon field, and two non-dynamical gauge fields $A_0^{(1,2)}$ in the adjoint of $\mathrm{U}(N+q)$ and $\mathrm{U}(N)$ respectively. The action is defined to be $S = \beta \int dt\, L$ with the lagrangian

$$L = \frac{1}{2}\,\mathrm{Tr}[(D_t\Phi)^\dagger D_t\Phi - U(|\Phi|^2)]\,, \tag{2.1}$$

where $D_t\Phi = \partial_t\Phi - iA_0^{(1)}\Phi + i\Phi A_0^{(2)}$ is the covariant derivative for the gauge group $\mathrm{U}(N+q) \times \mathrm{U}(N)$; and $U(|\Phi|^2) = -\frac{1}{2\alpha'}|\Phi|^2 + \cdots$ is the tachyon potential. In [9] it is proposed that the integer q corresponds to the background RR flux in 0A theory.

After integrating out the gauge fields and taking the double scaling limit, the system effectively reduces to N decoupled non-relativistic fermions moving on a plane, each with angular momentum q. The non-zero value of the angular momentum has the effect of pushing the particles away from the origin. The effective dynamics of the radial motion is then described by the Jevicki-Yoneya model [40] as shown in [21, 12]. This model is defined by the hermitean matrix model with the following potential (we always set $\alpha' = 1/2$ on matrix model side)

$$V(x) = -x^2 + \frac{M}{x^2}\,, \tag{2.2}$$

where the deformation parameter M is related[2] to the quantized flux q via $M = q^2 - 1/4$. The classical trajectory with the energy $-\mu$ is given by

$$x(\tau)^2 = \mu + \sqrt{M+\mu^2}\cosh(2\tau)\,. \tag{2.3}$$

This model is exactly solvable, so that one can compute non-perturbative scattering amplitudes explicitly as in [51, 21, 12] (see also [52, 53]).

It is also possible to compute the free energy in this model. One can see that the density of states $\rho(\epsilon)$ is given by [51, 52, 53]

$$\rho(\epsilon) = -\frac{1}{4\pi}\left[\psi\left(\frac{1}{2} - \frac{i\epsilon}{2} + \frac{q}{2}\right) + \psi\left(\frac{1}{2} + \frac{i\epsilon}{2} + \frac{q}{2}\right)\right] + \mathrm{const.}\,, \tag{2.4}$$

where $\psi(z) \equiv \frac{d\log\Gamma(z)}{dz}$. When the flux is large, $q \gg 1$, we can expand the expression (2.4) as follows[3]

$$\rho(\epsilon) \sim -\frac{1}{4\pi}\log(q^2+\epsilon^2) - \frac{1}{12\pi}\frac{q^2-\epsilon^2}{(q^2+\epsilon^2)^2} + \cdots. \tag{2.5}$$

[1] The electric brane couples to the symmetric combination of the RR 1-form potentials from the RR sector $(R+,R-) \oplus (R-,R+)$ of the theory, while the magnetic brane to the antisymmetric combination.

[2] This relation can be easily found using the second wave equation (10.8) in [9]. After redefining a new wave function $\psi = \lambda_i^{-\frac{1}{2}}\chi$, we get $-(\frac{d^2}{d\lambda_i^2} + \lambda_i^2 - \frac{q^2-1/4}{\lambda_i^2})\psi = E\psi$.

[3] To be exact a cut off L of the coordinate x is included in this expression as the extra term $\frac{1}{\pi}\log L$.

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Then we can compute the free energy at Fermi level $-\mu$.[4] In particular, when $|\mu| \ll q$, we get (see [53] for earlier discussions)

$$E = \int_{-\mu_0}^{-\mu} \epsilon\rho(\epsilon)d\epsilon \sim -\frac{1}{8\pi}q^2 \log q^2 + \frac{1}{24\pi}\log q^2 + \cdots, \tag{2.6}$$

where the dots denote analytic terms in q. The important point is that even if we take μ to zero, the perturbative expression is still non-singular due to non-zero value of q, as can be seen directly from eq. (2.6). At the point $\mu \sim q \sim 0$, the corresponding background of 0A theory describes the linear dilaton background, which is obviously singular. We conclude that there are two ways to stabilize the background: one is to add a Liouville potential in the worldsheet theory μe^{ϕ}, condensing the closed string tachyon field T, and another is to add large background flux q. In this paper we shall consider the latter case.

2.2 Collective field theory

In order to gain some insight into the dynamics of the model, it is useful to review briefly the collective field theory [54, 55, 4] of the Fermi liquid following [53]. In the classical limit, the collective motions of the liquid can be described in terms of a time dependent Fermi surface. For small perturbations, the Fermi surface consists of an upper and lower part, $p_\pm(x,t)$, subject to the equation of motion

$$\partial_t p_\pm(x,t) = x + \frac{M}{x^3} - p_\pm(x,t)\partial_x p_\pm(x,t)\,. \tag{2.7}$$

The static solution is given by

$$p_\pm(x)_{\rm static} = \pm\pi\phi_0(x) = \pm\sqrt{x^2 - \frac{M}{x^2} - 2\mu}\,, \tag{2.8}$$

with $-\mu$ being the Fermi level. The hamiltonian can be obtained in terms of the fields $p_\pm$ by integrating the single particle hamiltonian $h(p,x) = \frac{1}{2}p^2 + \frac{1}{2}V(x) + \mu$ over the Fermi sea

$$\begin{aligned} H &= \frac{1}{2\pi}\int dx \int_{p_-}^{p_+} dp h(p,x) \\ &= \frac{1}{2\pi}\int dx \left[\frac{1}{6}(p_+^3 - p_-^3) - \frac{1}{2}(x^2 - \frac{M}{x^2} - 2\mu)(p_+ - p_-)\right]. \end{aligned} \tag{2.9}$$

From the equation of motion and the hamiltonian, we can deduce the Poisson brackets

$$\begin{aligned} \{p_\pm(x), p_\pm(y)\} &= -2\pi\partial_x\delta(x-y)\,, \\ \{p_+(x), p_-(y)\} &= 0\,. \end{aligned} \tag{2.10}$$

We now change variables from x to the variable τ defined[5] by

$$d\tau = -\frac{dx}{\pi\phi_0(x)}\,, \tag{2.11}$$

[4] The Fermi level is measured from the top of the harmonic oscillator potential at $x = 0$.

[5] The turning point of the classical trajectory eq. (2.3) occurs at $x^2 = \mu + \sqrt{M + \mu^2}$ when $\tau = 0$, while as $\tau \to -\infty$ we have $x \to \infty$.

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and expand the fields $p_\pm$ about the static background

$$p_\pm = \pm\pi\phi_0 \pm \frac{1}{\pi\phi_0}\epsilon_\pm(\tau,t)\,. \tag{2.12}$$

In terms of these, we define a scalar field $\overline{S}(\tau,t)$

$$(2\pi)^{-1/2}\epsilon_\pm(\tau,t) = \pm\overline{\Pi}_S(\tau,t) - \partial_\tau\overline{S}(\tau,t)\,, \tag{2.13}$$

to obtain

$$H = \frac{1}{2}\int_{-\infty}^{0} d\tau\left[\overline{\Pi}_S{}^2 + (\partial_\tau\overline{S})^2 + \frac{e^{2\tau}}{\sqrt{M+\mu^2}}O(\overline{S}^3)\right] + E_0\,, \tag{2.14}$$

where

$$E_0 = -\frac{1}{3\pi}\int dx\left(x^2 - \frac{M}{x^2} - \mu\right)^{3/2}, \tag{2.15}$$

is the classical energy of the static background [53]. Thus, in the large τ region, we end up with a relativistic massless scalar field. As we approach the endpoint of the eigenvalue distribution at $\tau = 0$, non-relativistic self interactions of the scalar field $\overline{S}(\tau,t)$ become important. At the linearized level, the fields $\epsilon_\pm$ are simply the right and left moving modes satisfying $(\partial_t \pm \partial_\tau)\epsilon_\pm(\tau,t) = 0$. These modes are not independent but are mixed by Dirichlet boundary conditions at the endpoint of the eigenvalue distribution.[6]

The position dependent coupling constant scales as e^ϕ in string theory and $e^{2\tau}/\sqrt{M+\mu^2}$ in the matrix model. Thus for large M or small μ, we identify τ with the dimensionless linear dilaton spacetime coordinate[7] (asymptotically) as follows

$$\phi = -\frac{1}{2}\log M + 2\tau = -\log x^2\,. \tag{2.16}$$

The dominant interactions occur at the endpoint of the eigenvalue distribution near $\tau = 0$. At this point, the effective spacetime coupling constant is given by

$$g_{\text{eff}} \sim \frac{1}{\sqrt{M+\mu^2}}\,. \tag{2.17}$$

It remains small even at $\mu = 0$, when the deformation parameter M is large. Thus in this case, we expect the perturbation theory in the matrix model to remain valid, suggesting a weakly coupled string theory dual background with $g_{\text{eff}} \sim 1/q$.

When μ is large, we interpret scattering off the endpoint of the eigenvalue distribution in the matrix model as string scattering off the tachyon Liouville wall in spacetime. The Liouville wall prevents the strings from propagating into the strongly coupled region. Near the wall the strings interact with one another with effective coupling constant of order $1/\mu$ and they reflect back to the weakly coupled region. At $\mu = 0$, the tachyon background is

[6] We refer the reader in [53] for how to deal with technical difficulties arising from the boundary conditions at the endpoint of the distribution. As noted in [53], for the purposes of perturbation theory, one can keep only the right moving field, treating it as an independent field, and extend the range of the τ integration over the whole real line. One identifies $\epsilon_+(-\tau) = \epsilon_-(\tau)$.

[7] The coordinate ϕ is related to the spatial coordinate φ through $\phi = \sqrt{2/\alpha'}\,\varphi$.

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turned off and we interpret the scattering as tachyon scattering off a gravitational potential. The effective coupling constant is of order $1/q$. The corresponding background is curved. Indeed, as we argued earlier, the deformation parameter M in the matrix model is related to the number of units of RR 2-form flux in spacetime. The background RR flux curves spacetime. We will provide evidence that the relevant background at $\mu = 0$ is the extremal black hole solution of [43]. Effectively, the flux cuts off the strongly coupled region, 'hiding' it behind the horizon of the black hole.

The classical energy eq. (2.15) of the static background is infinite. We can regulate it by subtracting the energy of the corresponding background at $M = 0$ and imposing a long distance cut-off L on the coordinate x. Then at $\mu = 0$, we find for large M

$$E_0 = -\frac{M}{8\pi} \log \frac{M}{L^4} \,. \tag{2.18}$$

This is of course the leading order contribution to the free energy computation eq. (2.6). Using the relation to the linear dilaton spacetime coordinate ϕ, we can express the cut-off dependent divergent piece as $-M\phi/4\pi|_{\phi=-\infty}$. Clearly this cut-off corresponds to an infrared cut-off from the point of view of spacetime, regulating the infinite volume of spacetime. We shall see that the finite piece agrees precisely with the ADM mass of the extremal black hole background, which is obtained after we subtract the same divergent piece in the gravity theory.

From the point of view of the free energy computation, eq. (2.6), using the density of states $\rho(\epsilon)$, the cut-off corresponds to a cut-off of high negative energy modes deep in the Fermi sea

$$\mu_0 \sim L^2 \,. \tag{2.19}$$

In this large x region, the open string tachyon field has attained a large expectation value. In this sense, the cut-off corresponds to an ultraviolet cut-off in the open string theory on the D-branes. This is a manifestation of a UV/IR relation between the open and closed string sides of the duality similar to the UV/IR relation[8] in the AdS/CFT correspondence [58, 59]. A probe fermion eigenvalue following the classical Fermi trajectory corresponds to a D0 brane/anti-D0 brane pair decaying into closed string radiation [5, 6]. In the large x region, the fermion is relativistic and can be bosonized. It describes a coherent state of closed strings. In the open string channel, this state involves high frequency off-shell open strings [60, 61].

An important result from the old studies of this matrix model is the fact that scattering amplitudes involving an odd number of scalars $\overline{S}$ vanishes [40, 53, 51]. In the perturbative regime, this follows from the fact that the cubic interaction vertex at $\mu = 0$ vanishes on shell. This suggests that there should be a field redefinition under which the S-matrix remains invariant and makes a $\mathbb{Z}_2$ symmetry $\overline{S} \leftrightarrow -\overline{S}$ in the effective action manifest. From the point of view of spacetime physics, the collective scalar field $\overline{S}$ describing the fluctuations of the Fermi sea is related to the closed string tachyon field T. The spacetime effective action for the tachyon field has such a symmetry. Under this symmetry the action

[8] See [56, 57] for a similar relation in the context of two dimensional bosonic string theory.

remains invariant if one also interchanges the electric and magnetic RR one-form potentials. From the point of view of the worldsheet theory, this operation is $(-1)^{F_L}$ where F_L is the worldsheet fermion number:

$$(-1)^{F_L}: \quad \begin{array}{c} T \longleftrightarrow -T \\ F^{(e)} \longleftrightarrow F^{(m)} \,. \end{array} \tag{2.20}$$

In the presence of a tachyon background $T = \mu e^{\phi}$ this symmetry is broken spontaneously. Depending on the sign of μ only electric or magnetic branes are allowed physical states and so only one type of flux can be induced. For large positive μ, the dual matrix model describes the dynamics of $N + q$ D0 and N anti-D0 electric branes.[9] At $\mu = 0$, we can add q magnetic branes. From the spacetime point of view, there are q branes of each type localized at $\phi = \infty$ because of the linear dilaton background. Their effect is to induce q units of electric and q units of magnetic 2-form flux in spacetime.[10] The corresponding black hole has two types of equal charges, and the effective action for the closed string tachyon is invariant under the symmetry (2.20).

Finally, let us comment on the relation between the collective scalar field $\overline{S}$ and the spacetime tachyon field T. Their relation is non-local; in momentum space, they are related by a momentum dependent phase, the leg-factor. To find the leg factor, we follow the method presented in [2, 51]. Using the operator proposed in [8]

$$\lim_{l \to 0} \int dt\, e^{iPt}\, \mathrm{Tr}\, e^{-l\bar{\Phi}\Phi}\,, \tag{2.21}$$

we get the following leg factor phase

$$e^{i\delta(P)} = (q^2 + \mu^2)^{-i\frac{\sqrt{\alpha'}}{2\sqrt{2}}P}\, \frac{\Gamma(i\sqrt{\alpha'/2}\, P)}{\Gamma(-i\sqrt{\alpha'/2}\, P)}\,. \tag{2.22}$$

The operator with finite l creates a macroscopic loop with length l in the dual Riemann surface. Here we take the limit $l \to 0$ to realize a closed string insertion. These leg factors have poles at imaginary integer valued momenta corresponding to resonances in the euclidean amplitudes due to extra discrete states in the theory [2]. These discrete states are remnants of oscillator modes of the string.

2.3 The extremal black hole and the correspondence

Now let us consider vacuum solutions of 0A string theory in the presence of RR flux. The effective low energy action[11] is given by [9]

$$\begin{aligned} S_{2d} = \int d^2x\, \sqrt{-g}\Bigg[& \frac{e^{-2\phi}}{2\kappa^2}\left(\frac{8}{\alpha'} + R + 4(\nabla\phi)^2 - a(\nabla T)^2 + \frac{2a}{\alpha'}T^2 + \cdots\right) - \\ & - \frac{2\pi\alpha'}{4}\left(e^{-2T}|F^{(e)}|^2 + e^{2T}|F^{(m)}|^2\right) + q_+ F^{(e)} + q_- F^{(m)} + \cdots\Bigg]\,, \end{aligned} \tag{2.23}$$

[9] At large negative μ, we have a dual description in terms of magnetic branes.

[10] The flux background is stable against nucleation of brane anti-brane pairs because of the linear dilaton. The pairs cannot be separated and are confined within the horizon region of the extremal black hole.

[11] If we would like to have the action for the bosonic string, we must rescale $\alpha' \to \alpha'/2$.

where a is a certain constant. The familiar linear dilaton vacuum is given by

$$\phi = \sqrt{\frac{2}{\alpha'}}\varphi\,, \qquad g_{\mu\nu} = \eta_{\mu\nu}\,. \tag{2.24}$$

In this background, $q_\pm = 0$ and the field $\tilde{T} = e^{-\phi}T$ is massless. It corresponds to the scalar field $\overline{S}$ describing the collective motions of the Fermi liquid. The non-singular $\hat{c} = 1$ theory is obtained by condensing the tachyon field $T = \mu e^{\sqrt{2/\alpha'}\,\varphi}$, which is proposed to be dual to the 0A matrix model at the Fermi level $-\mu$ and $q = 0$ [9, 8].

Since we are interested in solutions invariant under (2.20) we consider background flux

$$F^{(e)} = F^{(m)}\,. \tag{2.25}$$

This flux can be induced by q electric and q magnetic D0-branes "at infinity". As can be seen from the action (2.23), there is no tachyon tadpole in this case. The tachyon background expectation value can be set to zero. The background flux generates a potential for the tachyon field $V_f(T) \sim q^2\cosh(2T)$. In the small T approximation, this is given by

$$V_f(T) \sim q^2(1 + 2T^2 + \cdots) = q^2(1 + 2e^{2\phi}\tilde{T}^2 + \cdots)\,. \tag{2.26}$$

The leading quadratic piece corresponds to a position dependent mass term for $\tilde{T}$, preventing propagation in the strongly coupled region. On the matrix model side, this is achieved by the deformation potential in (2.2).

As in massive type-IIA supergravity [62], the presence of RR-flux is equivalent to a cosmological constant. Indeed, integrating out $F^{(e)}$ and $F^{(m)}$ in (2.23), we obtain the following effective action (here, we set $2\kappa^2 = 1$ and $a = 1$):

$$S_{2d} = \int d^2x\,\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2 + c - (\nabla T)^2 + \frac{2}{\alpha'}T^2\right) + \Lambda(1 + 2T^2) + \cdots\right], \tag{2.27}$$

where $c \equiv 8/\alpha'$ and

$$\Lambda = -\frac{q^2}{2\pi\alpha'}\,. \tag{2.28}$$

The field equations obtained from the action (2.27) have the form:

$$\begin{aligned} R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi - \nabla_\mu T\nabla_\nu T &= 0 \\ R + c - (\nabla T)^2 + \frac{2}{\alpha'}T^2 + 4\nabla^2\phi - 4(\nabla\phi)^2 &= 0 \\ \nabla^2 T - 2\nabla\phi\nabla T + \left(\frac{2}{\alpha'} + 2\Lambda e^{2\phi}\right)T &= 0\,. \end{aligned} \tag{2.29}$$

A simple class of solutions in this theory can be obtained by considering a background with zero tachyon field, $T = 0$. In this case, the theory reduces to the dilaton gravity with negative cosmological constant studied in [43]. In particular, classical solutions in such theory correspond to black holes with mass parameter m (in this coordinate frame, the dilaton is given by ϕ):

$$ds^2 = -l(\phi)dt^2 + \frac{d\phi^2}{l(\phi)}\,, \tag{2.30}$$

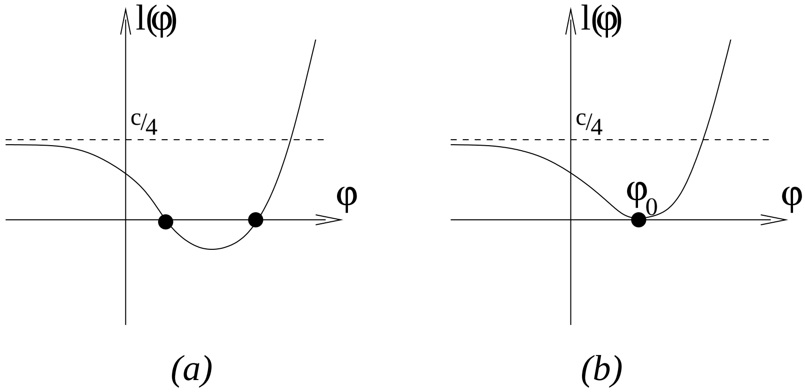

Figure 1: A plot of the conformal factor $l(\phi)$ for a non-extremal (a) and extremal (b) black hole.

where

$$l(\phi) = \frac{c}{4} - e^{2\phi}\left(\frac{\Lambda}{2}\phi + m\right). \tag{2.31}$$

These solutions describe Reissner-Nordstrom like charged black holes, and the strongly coupled region is hidden behind a horizon as in [41]. If we shift ϕ by a constant, we can still obtain a solution shifting (m, Λ) appropriately. In the weakly coupled region at $\phi \to -\infty$, the solution approaches the linear dilaton vacuum (2.24) and the metric is flat. In this region, the backreaction of the background flux becomes negligible. On the matrix model side this corresponds to the fact that the deformation term in the potential (2.2) is localized near $x = 0$ corresponding to the strongly coupling region.

The horizon corresponds to the zero of $l(\phi)$. When the mass parameter m is bigger than a critical value, $m > m_0$, where

$$m_0 = -\frac{\Lambda}{4} - \frac{\Lambda}{4}\log\left(-\frac{c}{\Lambda}\right), \tag{2.32}$$

there are two horizons as in the usual case of Reissner-Nordstrom black holes in four dimensions. When $m = m_0$ the inner and outer horizons coincide corresponding to the fact that the function $l(\phi)$ has a double zero at some point $\phi = \phi_0$, see figure 1. This is the extremal solution of [43]. The solution (2.30) has a naked singularity unless the mass parameter obeys the BPS like inequality $m \geq m_0$.

The location of the horizon of the extremal black hole occurs at

$$\phi_0 = \frac{1}{2}\log\left(-\frac{c}{\Lambda}\right), \tag{2.33}$$

when $l(\phi) = l'(\phi) = 0$. Thus the coupling constant near the horizon is of order $g_s \sim 1/q$, and the theory remains weakly coupled outside the horizon for large q. This was also the case from the analysis of the collective field theory of the Fermi liquid. On the matrix model side, the location of the horizon corresponds to the endpoint of the eigenvalue distribution. Thus we expect the Fermi sea to describe the region outside the horizon of the black hole. It is interesting to notice that the extremal black hole metric can be written in a simple form independent of Λ

$$l(\tilde{\phi}) = \frac{c}{4} + \frac{c}{2}e^{2\tilde{\phi}}\left(\tilde{\phi} - \frac{1}{2}\right), \tag{2.34}$$

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by shifting the relation between the dilaton and the spatial coordinate as

$$\tilde{\phi} = \phi - \frac{1}{2}\log\left(-\frac{c}{\Lambda}\right). \tag{2.35}$$

Asymptotically, this shifted coordinate corresponds to the variable τ in the matrix model.

For the extremal black hole solution the Hawking temperature vanishes. The scalar curvature goes to zero as $\phi \to -\infty$ and becomes of order one in string units near the horizon. In the near horizon limit, the geometry looks like two dimensional anti-de Sitter space [43]. In terms of a new space-like variable $u^{-1} = \sqrt{\frac{c}{2}}(\phi - \phi_0)$, the metric takes the standard form[12]

$$ds^2 = \frac{1}{u^2}(-dt^2 + R_{AdS}^2 du^2)\,, \tag{2.36}$$

where $R_{AdS} = \sqrt{2/c}$ is the radius of the anti-de Sitter space. Thus we end up with a stringy AdS_2. Since the curvature in this region is of order one in string units, stringy α' corrections become important and naively the low energy gravity approximation seems to break down. Unlike the case of [41], we do not have a full sigma model description of the background geometry.

Let us now obtain the ADM mass of the black hole. Using the standard formula in 2d gravity we get

$$M_0 = \frac{1}{2\sqrt{c}}(4m + \Lambda + 2\Lambda\phi)\Big|_{\phi=-\infty}\,. \tag{2.37}$$

For the extremal black hole $m = m_0$ is given by eq. (2.32). Hence, after subtracting the divergent cut-off dependent piece, we obtain for large q

$$M_0 = \frac{\Lambda}{2\sqrt{c}}\log\left(-\frac{\Lambda}{c}\right) = -\frac{q^2}{8\pi\sqrt{2\alpha'}}\log q^2\,. \tag{2.38}$$

This agrees precisely with the matrix model computation of the classical energy (2.18), where the same cut-off dependent piece is subtracted. Thus we would like to propose that the 0A matrix model with $\mu = 0$ and non-zero q is dual to 0A string theory on the extremal black hole background. More precisely, the matrix model includes stringy α' and g_s corrections to the low energy gravity analysis. Thus it describes a 'quantum' black hole. We believe that non-extremal black holes correspond to deformations of the 0A matrix model with Wilson lines[13] as in reference [42] leading to higher energy configurations.

One important evidence for this proposal is as follows. As we argued before, on the matrix model side, the scattering amplitude of an odd number of tachyon fields T vanishes [40, 53, 51]. On the gravity side, this just follows from the $\mathbb{Z}_2$ symmetry (2.20) of the theory in the symmetric flux background $F^{(e)} = F^{(m)}$.

[12] See [39] for a proposal for a matrix model dual of AdS_2 in two dimensional type 0A theory. For a further analysis of the classical solution see also [35].

[13] Here note that we have two different gauge fields in the matrix model side: diagonal (NS) and relative (R).

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2.4 Tachyon dynamics in the black hole background

Now, let us consider a weak tachyon field in the background of the extremal black hole (2.30)–(2.31). From (2.29) we find the equation of motion for the massless tachyon mode $\tilde{T} = e^{-\phi}T$,

$$l^2\tilde{T}'' + ll'\tilde{T}' + \left(ll' - l^2 + l\left(\frac{2}{\alpha'} + 2\Lambda e^{2\phi}\right)\right)\tilde{T} - \frac{\partial^2}{\partial t^2}\tilde{T} = 0 \tag{2.39}$$

where the prime denotes the derivative with respect to ϕ, which plays the role of the radial coordinate in the Schwarzschild gauge (2.30). Notice, that the coefficient of the linear derivative term vanishes if we write (2.39) in terms of the original spatial coordinate x, such that $\frac{d\phi}{dx} = l(\phi)$,

$$\frac{\partial^2}{\partial x^2}\tilde{T} + \left(ll' - l^2 + l\left(\frac{c}{4} + 2\Lambda e^{2\phi}\right)\right)\tilde{T} - \frac{\partial^2}{\partial t^2}\tilde{T} = 0\,. \tag{2.40}$$

Let us look at the tachyon mode with energy ω

$$\tilde{T} = \tilde{T}_\omega e^{-i\omega t}\,. \tag{2.41}$$

Substituting this into (2.40), we find that the tachyon equation of motion takes the form of the one-dimensional Schrödinger equation

$$\left(\frac{d^2}{dx^2} + \omega^2 - V_{\text{eff}}\right)\tilde{T}_\omega = 0\,, \tag{2.42}$$

with the effective potential

$$V_{\text{eff}} = -l\left(l' - l + \frac{c}{4} + 2\Lambda e^{2\phi}\right). \tag{2.43}$$

Using the explicit form (2.31) of the conformal factor $l(\phi)$ it is easy to see that V_{eff} vanishes in the asymptotic region, $\phi \to -\infty$, because the expression in brackets in (2.43) goes to zero in this limit. Similarly, since $l(\phi_0) = 0$ the effective potential vanishes at the horizon as well. At other points, V_{eff} is a positive function with a finite maximum of the order of $c^2 \sim (\alpha')^{-2}$. Notice that in the supergravity limit,[14] $\alpha' \to 0$, the height of the barrier in the effective potential goes to infinity, so that the tunneling from the asymptotic region, $\phi \to -\infty$, to the near-horizon region, $\phi \simeq \phi_0$, is highly suppressed. Thus low energy tachyons reflect off a barrier.

Taking backreaction into account, one would expect the resulting state to be a non-extremal black hole whose metric is given by (2.30) with $m > m_0$. In matrix model variables, the perturbation in the metric is of the order of $\delta m/x^4$, where δm is the energy above extremality. Let us now consider a pulse on the Fermi sea of width δx and height δp as in [4]. The analysis in [4] (see also [63]) shows that in order to produce a gravitational effect of order one we need $(\delta p)^2 \sim x^2$, i.e. the height of the pulse is comparable to the height of the whole sea. For such a large pulse, the tachyon self interaction is then

[14] Since we are using the effective field theory of two dimensional gravity, we do not trust the result beyond this low energy approximation.

also of order one and we cannot get into a situation where the gravitational interaction dominates.[15] So we cannot be sure that it is possible to make a non extremal black hole by sending in tachyon pulses from the asymptotic region as described in [43]. Perhaps the reflection amplitude in the matrix model can describe such non-extremal black hole states and their subsequent Hawking evaporation to the extremal black hole endpoint. It would be interesting to understand how in more detail. Notice also that the semiclassical gravity analysis breaks down in the near horizon region because the curvature is of order the string scale. Unfortunately, we do not have a full string sigma model to resolve this puzzle.

Let us now consider the reflection amplitude (or two point function) $S(P)$ (P is the momentum in the φ direction) in this background. In the fluctuation analysis around this background we can easily see that it includes the factor

$$S(p) \propto q^{-\sqrt{2\alpha'}iP}\,, \tag{2.44}$$

due to the shift eq. (2.35). Even though we cannot obtain the exact reflection amplitude in the background (2.30) and (2.34) due to the break down of low energy analysis, we may expect that the pole structure can be computable in the effective gravity theory. We can find an approximate expression by replacing $\tilde{\phi}e^{2\tilde{\phi}}$ with $e^{2\tilde{\phi}}$, which reduces the computation to the familiar one in the 2D black hole of [41]. Using the results[16] in [64], we find the momentum dependent factor

$$S(P) \propto \frac{\Gamma(i\sqrt{\alpha'/2}\,P)\,\Gamma^2(\frac{1}{2}-i\sqrt{2\alpha'}P)}{\Gamma(-i\sqrt{\alpha'/2}\,P)\,\Gamma^2(\frac{1}{2}-i\sqrt{\frac{\alpha'}{2}}P)}\,. \tag{2.45}$$

Now let us compare the results from spacetime computation (2.44) and (2.45) with the matrix model result (2.22), though we cannot expect the exact matching between them. The factor (2.44) in the spacetime analysis is indeed included in the matrix model result (2.22) when $\mu = 0$. The position[17] of poles in (2.45) also agrees with that in (2.22). These provide more supports of our proposal.

2.5 Matrix model thermodynamics

The finite temperature free energy of the 0A matrix model has been essentially computed in [53]. Up to one loop in the $1/q$ expansion, we have

$$F = -\frac{1}{8\pi}q^2 \log\left(\frac{q^2}{L^4}\right) + \frac{1}{24\pi}[1+(\pi T)^2]\log\left(\frac{q^2}{L^4}\right) + \cdots\,, \tag{2.46}$$

where T is the temperature and L is an infrared volume cutoff [53]. The temperature dependent piece shown is a one loop effect. The thermal entropy is given by

$$S = -\frac{\pi}{12}T\log\left(\frac{q^2}{L^4}\right) + \cdots\,. \tag{2.47}$$

This thermal ensemble corresponds to a gas of massless scalars on a line of size $\sim \log(L^4/q^2)$.

[15] The fermions are of course free. Tachyon interactions arise due to a strong dispersion in the pulse [4].

[16] Note that the convention $\alpha' = 2$ in bosonic string is equivalent to $\alpha' = 1$ in 0A string. We also have the relation $r = -2\phi$.

[17] Here we cannot expect that (2.45) does exactly agree with (2.22) because the former is a sort of a minisuperspace computation and the latter is an exact computation on the matrix theory side.

From the spacetime point of view, we interpret the thermal ensemble as a gas of tachyons in the asymptotic region of the extremal black hole background. Typically, such an ensemble would be unstable against gravitational collapse: one has a thermal energy density at temperature T and in infinite volume the total energy is infinite. However, in two dimensional string theory the linear dilaton term in the worldsheet action implies that the gravitational coupling vanishes exponentially in the asymptotic region and so the backreaction of the thermal gas may be negligible. Thus we would expect that the higher loop contributions to the free energy are well behaved, as it is suggested by the matrix model computations of [53].

Notice also that the euclidean geometry of the extremal black hole is smooth and covers the region outside the horizon. If we periodically identify asymptotic time at $\phi = -\infty$ with period $2\pi\beta$, we still obtain a smooth geometry without any conical singularity. As we move toward the stringy AdS_2 region, the effective temperature increases and becomes of order the string scale. At this point, the gravity analysis should be replaced with either the full string theory or the dual matrix model.

Of course, one would like to understand if non-extremal black holes are states within the matrix model. These thermal states have an extra contribution to the free energy of order $\delta F \sim q^2 T$. Unlike eq. (2.46) this looks like a genus zero contribution. Thus at large q, we do not expect to see a phase transition in the perturbative regime we explored.[18] Instead we can speculate that these states can be understood as deformations of the matrix model by winding modes as in [42].

3. Type 0B string with RR-flux

Here we would like to consider the T-dual of the 0A background with RR-flux $q \neq 0$ discussed in the previous section. Consider the 0A theory compactified on a euclidean time circle of radius R. In this theory one can define two Wilson line operators with winding number n by $\Omega^n_\pm = \mathrm{Tr}(e^{in\int A^{(1)}}) \pm \mathrm{Tr}(e^{in\int A^{(2)}})$, where $A^{(1,2)}$ denote the gauge fields on branes and anti-branes respectively. These Wilson lines correspond to the winding modes in 0A string theory. After T-duality they become momentum modes, which describe fluctuations of the two Fermi seas in the 0B model. Similarly, the Fermi sea fluctuation modes in the original 0A theory are T-dual to the Wilson lines in the 0B model.[19]

We wish to apply the T-duality transformation to the RR-flux background discussed in the previous section. First, let us consider T-duality applied to the euclidean continuation of the black hole solution (2.30). We obtain the following metric

$$ds^2 = \frac{d\theta^2 + d\phi^2}{l(\phi)}\,, \tag{3.1}$$

where θ is the T-dual time direction. Similarly, the string coupling is given by

$$g_s = \frac{e^\phi}{\sqrt{l(\phi)}}\,. \tag{3.2}$$

[18]We thank S. Minwalla for many useful discussions on these and related issues.

[19]The detailed structure of these T-duality transformations is studied independently in [65].

In the region $\phi \simeq \phi_0$, we still find an AdS_2 geometry except now we approach the boundary of the space. This region seems to be strongly coupled, which naively contradicts the fact that the original 0A description is non-singular. As we shall see below, gravitational effects prevent particles from entering the strongly coupled region. Note, that in the present case the situation is different from the T-dual description of the usual black hole [41]. The latter theory is described by the sine-Liouville theory, which implies a closed string tachyon condensate. In our case there is no tachyon condensation, as can be seen directly from the $T \leftrightarrow -T$ symmetry of the background configuration.

Now, let us consider the T-duality action on the RR fields. Under T-duality, the non-dynamical fields $F_{01}^{(e)}$ and $F_{01}^{(m)}$ in 0A theory transform into spatial components of the 'electric' and 'magnetic' 1-form fields F and $*F$, which are dynamical in the 0B theory (locally, we can write $F = dC$). In other words, we conclude that the 0B dual of the flux background (2.25) can be described by the self-dual (or anit-self-dual) flux configuration

$$F_+ = f\,, \qquad F_- = 0\,. \tag{3.3}$$

where $F_\pm$ are the light-like components of the flux F. Such configurations are similar to instanton solutions in four-dimensional gauge theory. As we shall see below, the flux configurations (3.3) play a special role in 0B theory as well. In particular, they automatically satisfy the tachyon tadpole condition.

Before we proceed to describing specific solutions in 0B theory with RR flux, we point out that the non-constant modes of the flux $F_- \sim f_n e^{in(t-\phi)}$ correspond to Ω_-^n. While generic deformations by Ω_-^n may lead to charged non-extremal black hole states, cf. [42], here we will be mainly interested in deformations by constant RR-flux.

3.1 Fermi sea of type 0B with RR flux and $|\mu| > f$

Next let us consider the 0B Fermi sea dynamics using the formalism of [4, 55]. The ground state of the two Fermi seas (left $x < 0$ and right $x > 0$) in 0B model with $\mu > 0$ is defined by

$$p_\pm^l = \pm\sqrt{x^2 - 2\mu}\,, \qquad p_\pm^r = \mp\sqrt{x^2 - 2\mu}\,. \tag{3.4}$$

One can parameterize the fluctuations of each Fermi sea as follows

$$p_\pm^l = \mp x \pm \frac{\epsilon_\pm^l}{x}\,, \qquad p_\pm^r = \mp x \pm \frac{\epsilon_\pm^r}{x}\,. \tag{3.5}$$

If we use the spatial coordinate ϕ, such that $e^{-\phi} \sim x^2$, then we can define two collective fields S_{NS} and S_R which in the asymptotic region $|x| \gg 1$ are given by

$$\epsilon_\pm^l + \epsilon_\pm^r \sim \pm\partial_t S_{NS} - \partial_\phi S_{NS}\,, \qquad \epsilon_\pm^l - \epsilon_\pm^r \sim \pm\partial_t S_R - \partial_\phi S_R\,. \tag{3.6}$$

The canonical ground state corresponds to $S_{NS} = -2\mu\phi$ and $S_R = 0$, which is obtained by adding the Liouville term $\int \mu\phi e^\phi$ in the 0B worldsheet theory.

Now we can consider perturbations by adding RR flux F in spacetime. We consider[20] the case of constant RR field strength $F_\phi = 2f$, which is equivalent[21] to $S_R = 2f\phi$ (S_R corresponds to the RR potential C). Thus this configuration is described by shifting the Fermi levels on each side of the potential asymmetrically. In the asymptotic region we obtain

$$p^l_\pm \sim \mp x \pm \frac{\mu - f}{x}, \qquad p^r_\pm \sim \mp x \pm \frac{\mu + f}{x}. \tag{3.7}$$

For large positive μ, large enough so that $\mu > |f|$, the Fermi sea is described in phase space as in figure 2. Even though this flux background is perturbatively stable, it will eventually decay due to non-perturbative effects (D-instantons) to a state for which the Fermi levels on the two sides are equal. When $|\mu| \gg |f|$, such tunneling effects can be neglected. The asymmetric Fermi sea takes a very long time to decay in this case.

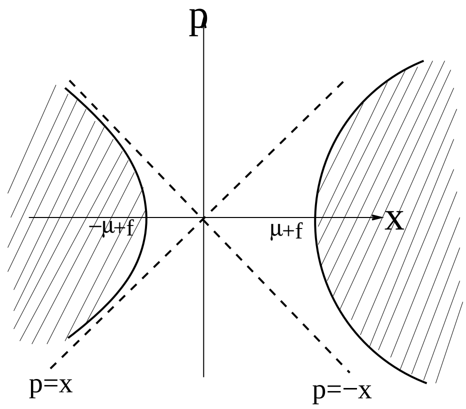

Figure 2: The Fermi sea for the 0B flux background when $\mu > f > 0$.

When we come to the point $|\mu| = |f|$, the Fermi sea includes the strongly coupled region $|x| \sim 0$, and non-perturbative effects become important. The situation is similar to the case without flux and $\mu = 0$. One however can go over the barrier at $\mu = 0$ by considering negative μ and obtain a dual perturbative description setting $T \leftrightarrow -T$ in spacetime [9]. Thus we may expect that we can go over the barrier $|\mu| = |f|$ without encountering a phase transition. As we will argue later, this is indeed the case. For this purpose let us examine the partition function (free energy) in the next subsection.

3.2 Free energy and T-duality

We can also compute the perturbative free energy[22] or partition function. It is defined by [53, 42, 9]

$$\frac{\partial^2 \Gamma(\mu)}{\partial \mu^2} = \rho(\mu). \tag{3.8}$$

Using this definition, we get the 0A model partition function ($\mathcal{Z} \equiv 2\pi R \Gamma(\mu)$)

$$\mathcal{Z}_A = 2\,\mathrm{Re}\Bigg[-R\left(\frac{\mu}{2} + \frac{iq}{2}\right)^2 \log\left(\frac{\mu}{2} + \frac{iq}{2}\right) - \frac{1}{24}\log\left(\frac{\mu}{2} + \frac{iq}{2}\right)\left(2R + \frac{1}{2R}\right) + \\ + \sum_{m=1}^{\infty} \left(\frac{\mu}{2} + \frac{iq}{2}\right)^{-2m} f_m(2R)\Bigg]. \tag{3.9}$$

[20] Note that the RR vertex operator has the Liouville dressing e^{ϕ}. The RR field strength $F = dC$ is defined by multiplying with $e^{-\phi}$ so that we obtain a massless field as in the NSNS sector.

[21] To be exact we need to take into account the leg factor. However in our example that is given by a constant and can be absorbed in f.

[22] More precisely, this is a Legendre transformation of the usual free energy with a fixed fermion number.

We have defined the function f_m as

$$f_m(y) = (2m-3)!\,2^{-2m}\sum_{k=0}^{m} R^{-2k}\frac{|2^{2(m-k)}-2||2^{2k}-2||B_{2(m-k)}||B_{2k}|}{2(m-k)!2k!}\,, \tag{3.10}$$

where B_{2m} are the Bernoulli numbers ($B_{2m} = (-1)^{m-1}|B_{2m}|$). Notice also the property

$$f_m(y) = y^{-2m} f_m\left(\frac{1}{y}\right). \tag{3.11}$$

On the other hand, one can also compute the 0B partition function (at radius $\tilde{R}$) with RR flux f by considering the two Fermi sea picture with Fermi levels $\tilde{\mu}+f$ and $\tilde{\mu}-f$ as follows

$$\mathcal{Z}_B = -\frac{\tilde{R}}{4}(\tilde{\mu}+f)^2\log(\tilde{\mu}+f)^2 - \frac{1}{48}\log(\tilde{\mu}+f)^2\left(\tilde{R}+\frac{1}{\tilde{R}}\right) + \sum_{m=1}^{\infty}(\tilde{\mu}+f)^{-2m} f_m(\tilde{R}) + (f \leftrightarrow -f)\,. \tag{3.12}$$

Then, it is easy to see that $\mathcal{Z}_A = \mathcal{Z}_B$ if we set[23]

$$\tilde{\mu} = \mu R\,, \qquad f = iqR\,, \tag{3.13}$$

and use the T-duality relation $\tilde{R} = 1/2R$ at $\alpha' = 1/2$. The relation (3.13) is very natural from the following observation. If we take time-like T-duality of type 0A theory, we generally get a different theory called 'type 0B*' theory from the usual type 0B theory in the same way as we get type-II* theory [67]. The difference between 0B and 0B* appears in the Ramond sector and the latter has the wrong sign in front of kinetic term of RR-fields. To map the RR-flux in 0B* to 0B we have to multiply it by i. This explains the correspondence described above.

3.3 Type 0B with RR flux and $|f| \geq |\mu|$

Let us return to the original problem: the interpretation of the background with large RR flux $|f| \geq |\mu|$. Let us first consider the case $\mu = 0$ and $f > 0$. Naively the double Fermi sea picture looks ill-defined. Actually there should be no problem in understanding such a background since the type 0B theory is non-perturbatively well defined [8, 9]. Physical quantities such as scattering amplitudes and the partition function remain finite. To understand such a background non-perturbative corrections may turn out to be important. The best way to analyze this case is to examine the non-perturbative expressions for the partition function and amplitudes. Interestingly even if we set $\mu = 0$, the partition function remains well behaved having a perturbative expansion with respect to f^{-1} as can be seen from (3.12) . One naturally expects this from the point of view of T-duality from the 0A case, where the partition function at level $\mu = 0$ has a perturbative expansion in terms of $1/q$. Scattering amplitudes also have the same property [12, 13].

Now, if we take the T-dual[24] of the 0A flux background $\mu = 0$ and $q > 0$, we must add the flux (3.3) with only one light-like component F_+ (or equally $F_t = F_\phi$). Indeed at $\mu = 0$

[23] A similar result can be also found in $c = 0$ matrix model [66].

[24] To be exact we have the 0B* theory as T-dual of 0A and furthermore we have to consider its continuation to 0B theory. As we will discuss later this is indeed possible.

we can add both components F_ϕ and F_t [9]. This is the dual of having both $F^{(e)} = F^{(m)}$ in the 0A model. In this case we have non-zero expectation values only for $\epsilon_-^{l,r}$ in (3.6). Naively then we get a Fermi sea which does not correspond to a static background in the two dimensional gravity theory at low energies. The Fermi levels on the left and the right do not match near $x = 0$ and this region should be included in the occupied phase space. So one would expect a time dependent configuration to result. However, we expect that the relevant background is time independent as in the 0A dual picture.

It is easy to obtain a static configuration by a small modification of the previous configuration with only $\epsilon_-^{l,r}$. This is given by the two Fermi seas $FS1$ and $FS2$ defined by (see figure 3)

$$\begin{aligned} FS1 &= \left\{(x,p)|x < -\sqrt{p^2+f}\right\}, \\ FS2 &= \left\{(x,p)|p \geq -x,\, p < \sqrt{x^2+f}\right\}. \end{aligned} \quad (3.14)$$

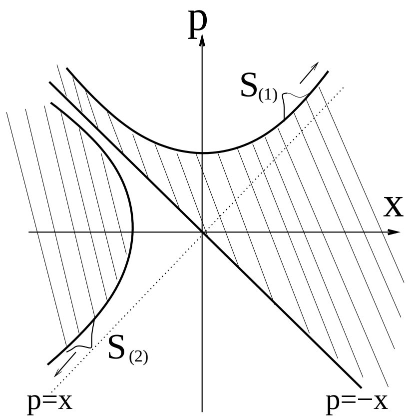

Figure 3: The Fermi sea for the 0B flux background when $\mu = 0$ and $f > 0$.

The important property of this configuration is that it is invariant under the following $\mathbb{Z}_2$ transformation[25] defined in [2, 9]: transform (x,p) into $(-p,-x)$ and replace a fermion with a 'hole'. From the closed string point of view, this $\mathbb{Z}_2$ transformation acts as

$$(-1)^{F_L}: \quad \begin{aligned} T &\longleftrightarrow -T \\ F_\pm &\longleftrightarrow \pm F_\pm\,. \end{aligned} \quad (3.15)$$

The compact RR scalar C is dualized by this action.

Let us consider a spacetime interpretation of this configuration. We will provide technical details on this in the next subsection. It is useful to note that the axion C is compact at the self-dual radius [9] and is indeed invariant under the $\mathbb{Z}_2$ symmetry. Then we can introduce two other scalar fields by using the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ current algebra

$$\cos(C^{(L,R)}) = \partial_\pm \tilde{C}^{(L,R)}, \qquad \sin(C^{(L,R)}) = \partial_\pm \tilde{\tilde{C}}^{(L,R)}. \quad (3.16)$$

Notice that $\tilde{C}$ is invariant under the symmetry (3.15), while $\tilde{\tilde{C}}$ is not. Now, from this fact there is an important possibility that the required modification corresponds to adding the flux $\tilde{F}_- = \partial_- \tilde{C}$, in addition to F_+. As we will see in the next subsection, this can be realized by considering a mixed state[26] of particles and holes, which can be approximated by the Fermi sea (3.14) in the large f limit. This configuration represents a type 0B background with RR fluxes $F_+ = \tilde{F}_- = f$ and no tachyon. Notice that since it is invariant under the

[25]In the same way we can also define another $\mathbb{Z}_2$ transformation by $(x,p) \to (p,x)$ and particle-hole exchange. This corresponds to another $\mathbb{Z}_2$ action $T \to -T,\ F_\pm \to \mp F_\pm$.

[26]The terminology 'mixed state' refers to a state involving superposition of particle and hole creation operators (see section 3.4).

$\mathbb{Z}_2$ symmetry, the tachyon tadpole is zero and we can set $\mu = 0$ consistently. It is also possible to show that the corresponding classical solution in the effective gravity theory is indeed static as we will see in the next section.

On the other hand, if we consider an exact pure state represented by (3.14), this should be a rather complicated quantum state of D-branes which cannot be approximated in the classical effective gravitational theory. This can be seen from the fact that with only the flux F_+ turned on, then we cannot construct static classical solutions as is obvious from the later analysis in section 4.

The difference between this Fermi sea and the usual Fermi sea at level $-\mu$ without flux is that it includes the strongly coupled region near $x \sim 0$. Naively, this means that in this case non-perturbative effects (D-instantons) are important. However, as we argued before, the perturbative region is well decoupled from the strongly coupled region when f is large. Thus we may neglect non-perturbative effects for large f. Below we will discuss the perturbative behavior.

In the case at hand, we have two collective scalar fields $S_{(1)}$ and $S_{(2)}$ describing the fluctuations of the two Fermi seas $FS1$ and $FS2$ respectively.[27] It is useful to define $S_{(\pm)} = S_{(1)} \pm S_{(2)}$ by taking the corresponding linear combinations. We can identify these with the spacetime fields as follows. Since any scattering amplitude of an odd number of fields $S_{(-)}$ vanishes in the matrix model, it must be identified with the tachyon field T, which is also odd under the symmetry (3.15). On the other hand, the field $S_{(+)}$ is invariant under the $(-1)^{F_L}$ and we can identify it with the RR scalar $C^{(L)} + \tilde{C}^{(R)}$. Thus we obtain

$$S_{(-)} = Te^{-\phi}\,, \qquad S_{(+)} = C^{(L)} + \tilde{C}^{(R)}\,. \tag{3.17}$$

Furthermore, we argue that the collective field theory is essentially the same as the usual 0B model (with no flux) setting $\mu = f$. The perturbative amplitudes are the same as in usual 0B model at non-zero μ and no flux except that we must replace f with μ. The partition function is indeed given by after the same replacement (see (3.12)).

It will not be simple to derive the perturbative amplitudes on the string theory side because the linear dilaton theory is perturbed not only by the RR-vertex operator for F_+ but also by the operator for $\tilde{F}_-$. The latter cannot be written in the RNS formalism in a simple way. The Green-Schwarz formalism may be required to work out the amplitudes. We leave this as a future exercise. However, we would like to point out that it would not be so surprising to get the same results as the usual $N = 1$ Liouville theory. Indeed in the linear dilaton theory the free field correlation functions of NSNS and RR-sector vertex operators remain essentially the same (up to leg factors) under the exchange of NSNS ones with RR ones as shown in [68].

Finally we would like to mention that it is straightforward to generalize this to the case $|f| > |\mu| > 0$. It corresponds to the 0B background with fluxes $F_+ = \tilde{F}_- = f$ and tachyon $T = \mu e^{\phi}$. The Fermi surface is given by replacing f with $f + \mu$ for $FS1$ and with $f - \mu$ for $FS2$.

[27] The Fermi sea $FS2$ has another Fermi surface at $p = -x$. However, we do not consider the corresponding collective field because due to tunneling effects this surface is highly smeared. Part of it is hidden inside the strongly coupled region.

3.4 Fermi sea picture of SU(2) rotation

When the tachyon background field is zero $\mu = 0$, the axion field C is compact at the self-dual radius so that the theory is S-dual (electro-magnetic dual) at this point [9]. If we neglect non-perturbative effects, the theory is invariant under the shift symmetry $C \to C+\alpha$.[28] The left and right moving currents are given by $J^{3(L)} = \partial_+ C$ and $J^{3(R)} = \partial_- C$ (see also [12]). Actually these are just the components of the RR flux F_+ and F_- respectively. Thus in the asymptotic region, we can define new scalar fields $\tilde{C}$ and $\tilde{\tilde{C}}$ by the following SU(2) rotation of the current algebra

$$J^1_{(L,R)} = \cos(C_{(L,R)}) = \partial_\pm \tilde{C}_{(L,R)}\,, \qquad J^2_{(L,R)} = \sin(C_{(L,R)}) = \partial_\pm \tilde{\tilde{C}}_{(L,R)}\,. \tag{3.18}$$

We would like to discuss the matrix model interpretation of this SU(2) symmetry. Note that this symmetry is manifest in the asymptotic region. Since we take the linear dilaton background as the ground state, corresponding to setting $\mu = 0$ in the matrix model, we define the creation operator of a hole $a_i^{(1,2)\dagger}$ and that of a particle (fermion) $b_i^{(1,2)\dagger}$ as follows (see figure 4)

$$\begin{aligned} a_i^{(1,2)\dagger} &= \psi^{(1,2)}_{\epsilon_i} \quad (\epsilon_i < 0)\,, \\ b_i^{(1,2)\dagger} &= \psi^{(1,2)\dagger}_{\epsilon_i} \quad (\epsilon_i > 0)\,, \end{aligned} \tag{3.19}$$

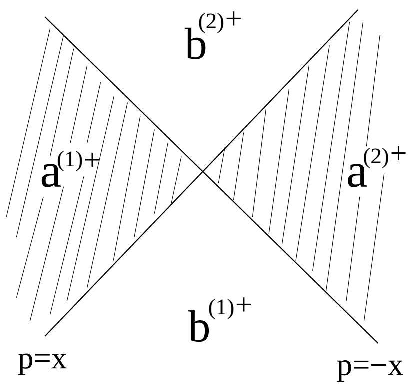

Figure 4: The particle and hole creation operators.

where $(1,2)$ denotes each of the two Fermi seas on the left and right. ϵ_i is the energy of each fermion labeled by i, which runs from 1 to N. We can define the annihilation operators in a similar way. Fluctuations of the Fermi surface are described by correlated particle/hole pairs.

Let us consider those operations which preserve the total energy. These will turn out to be the required symmetries. We are interested in only linear transformations of $a_i^{(1,2)\dagger}$ and $b_i^{(1,2)\dagger}$. (Below we omit the index i for simplicity.) Since these four complex fermions have the same energy ϵ_i, we get a U(4) symmetry. This is divided into $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, which corresponds to the decomposition into left and right parts. One U(1) factor included in U(4) amounts to shifting the phase of all fermions on both sides of the Fermi sea in the same way. It corresponds to the shift of tachyon field $T_{L,R}$ as can be seen from the bosonization formula $\psi^{(1)} = e^{i\frac{T+C}{2}}$, $\psi^{(2)} = e^{i\frac{T-C}{2}}$. The remaining $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry is identified with the SU(2) rotation of RR field.

The $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry is realized as follows. Define the vectors $u_1 = (a^{(1)}, b^{(1)})$, $u_2 = (b^{(2)}, a^{(2)})$, $v_1 = (a^{(1)}, b^{(2)})$ and $v_2 = (b^{(1)}, a^{(2)})$. The elements of $\mathrm{SU}(2)_L$ ($\mathrm{SU}(2)_R$) act on u_1 and u_2 (v_1 and v_2) in the fundamental representation. In particular their U(1) subgroups shift the phases

$$\mathrm{U}(1)_L: \; a^{(1)\dagger} \to e^{i\theta} a^{(1)\dagger}\,, \quad b^{(1)\dagger} \to e^{-i\theta} b^{(1)\dagger}\,, \quad a^{(2)\dagger} \to e^{-i\theta} a^{(2)\dagger}\,, \quad b^{(2)\dagger} \to e^{i\theta} b^{(2)\dagger}\,, \tag{3.20}$$

$$\mathrm{U}(1)_R: \; a^{(1)\dagger} \to e^{i\theta} a^{(1)\dagger}\,, \quad b^{(1)\dagger} \to e^{i\theta} b^{(1)\dagger}\,, \quad a^{(2)\dagger} \to e^{-i\theta} a^{(2)\dagger}\,, \quad b^{(2)\dagger} \to e^{-i\theta} b^{(2)\dagger}\,. \tag{3.21}$$

[28] Non-perturbatively, this symmetry is violated by D-instanton effects and becomes discrete. Since this discrete shift symmetry is a gauge symmetry, the axion field is compact.

These U(1) subgroups correspond to the shift of axion $C_{L,R}$ in left and right-moving sectors, respectively. They shift the relative phase of particles on each side of the Fermi sea.

The spacetime T-dualities are defined by $g_L : (C_L, C_R) \to (-C_L, C_R)$ and $g_R : (C_L, C_R) \to (C_L, -C_R)$. The action of g_L (g_R), which is included in the original U(4) symmetry, is given by the exchange of $a^{(1,2)\dagger}$ ($a^{(1,2)\dagger}$) with $b^{(1,2)\dagger}$ ($b^{(2,1)\dagger}$). Note that g_R is exactly the same as $(-1)^{F_L}$ defined by (3.15).

The full SU(2) generators are given by

$$J_L^a = u_1^\dagger T^a u_1 + u_2^\dagger T^a u_2 \,, \qquad J_R^a = v_1^\dagger T^a v_1 + v_2^\dagger T^a v_2 \,, \tag{3.22}$$

where T^a are the Pauli matrices. We choose a such that $a = 3$ corresponds to the U(1) subgroup realizing the shift symmetry of the axion field. For example, $J^3_{L,R}$ is given by[29]

$$\begin{aligned} J_L^3 &= a^{(1)\dagger}a^{(1)} - a^{(2)\dagger}a^{(2)} - b^{(1)\dagger}b^{(1)} + b^{(2)\dagger}b^{(2)} \,, \\ J_R^3 &= a^{(1)\dagger}a^{(1)} - a^{(2)\dagger}a^{(2)} + b^{(1)\dagger}b^{(1)} - b^{(2)\dagger}b^{(2)} \,. \end{aligned} \tag{3.23}$$

It is easy to see that if we want to have non-zero expectation values for these, we have to deform the Fermi sea on the left and right in an asymmetric way. These currents, attaining non zero expectation values, describe the RR flux backgrounds $F_\pm \neq 0$. This is consistent with the identification (3.6).

Now we are interested in the other generators $J^{1,2}$, which correspond to the 'exotic RR fields' $\tilde{C}$ and $\tilde{\tilde{C}}$. They are explicitly given by

$$\begin{aligned} J_L^1 &= a^{(1)\dagger}b^{(1)} + b^{(1)\dagger}a^{(1)} + b^{(2)\dagger}a^{(2)} + a^{(2)\dagger}b^{(2)} \,, \\ J_L^2 &= -ia^{(1)\dagger}b^{(1)} + ib^{(1)\dagger}a^{(1)} - ib^{(2)\dagger}a^{(2)} + ia^{(2)\dagger}b^{(2)} \,, \\ J_R^1 &= a^{(1)\dagger}b^{(2)} + b^{(2)\dagger}a^{(1)} + b^{(1)\dagger}a^{(2)} + a^{(2)\dagger}b^{(1)} \,, \\ J_R^2 &= -ia^{(1)\dagger}b^{(2)} + ib^{(2)\dagger}a^{(1)} - ib^{(1)\dagger}a^{(2)} + ia^{(2)\dagger}b^{(1)} \,. \end{aligned} \tag{3.24}$$

We argue that the SU(2) rotated field strengths correspond to the expectation values of these currents $F_\pm = \langle J^3_{L,R}\rangle$, $\tilde{F}_\pm = \langle J^1_{L,R}\rangle$ and $\tilde{\tilde{F}}_\pm = \langle J^2_{L,R}\rangle$ up to a constant. Notice that this identification is uniquely determined once we fix the interpretation of U(1) subgroup (shift of C). Under the two T-dualities the currents are rotated as follows $J^1 \to J^1$, $J^2 \to -J^2$, and $J^3 \to -J^3$ for the left and right sectors respectively.

We can show the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ algebra

$$[J_L^a, J_L^b] = 2i\epsilon^{abc}J_L^c \,, \qquad [J_R^a, J_R^b] = 2i\epsilon^{abc}J_R^c \,, \qquad [J_L^a, J_R^b] = 0 \tag{3.25}$$

explicitly. Let us denote the linear dilaton background $\mu = f = 0$ by $|0\rangle$. Then it is trivial to see that

$$J^a_{L,R}|0\rangle = 0. \tag{3.26}$$

[29]We can also define the current which corresponds to the tachyon field shift as $J^0 = a^{(1)\dagger}a^{(1)} + a^{(2)\dagger}a^{(2)} - b^{(1)\dagger}b^{(1)} - b^{(2)\dagger}b^{(2)}$. This is proportional to the fermion number operator. The expectation value of $\langle J^0\rangle$ is proportional to μ. This generator changes its sign by the T-duality action $g_{L,R}$. The hamiltonian is given by $H = a^{(1)\dagger}a^{(1)} + a^{(2)\dagger}a^{(2)} + b^{(1)\dagger}b^{(1)} + b^{(2)\dagger}b^{(2)}$.

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Next let us consider the Fermi sea of figure 3. On a first guess, we may conclude that it describes a state such that only $\langle J^3_L\rangle$ (i.e. F_+) is non-zero. Naively this seems to contradict our previous conjecture that the configuration of figure 3 can be regarded as a background with F_+ and $\tilde{F}_-$. However, as we will argue below, we can also consider a mixed state with the right property. Note also that since this configuration is invariant by the action g_R, the tachyon background field is set to zero.

Thus it will be very interesting to ask what kind of configuration of the Fermi sea has a non-zero value of the RR flux $\tilde{F}_-$ (or $\langle J^1_R\rangle$). To find the answer, we define a new basis of creation operators

$$c^{(1)\dagger}_{\pm} = \frac{1}{\sqrt{2}}(a^{(1)\dagger} \pm b^{(2)\dagger})\,, \qquad c^{(2)\dagger}_{\pm} = \frac{1}{\sqrt{2}}(a^{(2)\dagger} \pm b^{(1)\dagger})\,. \tag{3.27}$$

Then in this basis J^1_R is diagonal

$$J^1_R = \left(c^{(1)\dagger}_{+}c^{(1)}_{+} + c^{(2)\dagger}_{+}c^{(2)}_{+} - c^{(1)\dagger}_{-}c^{(1)}_{-} - c^{(2)\dagger}_{-}c^{(2)}_{-}\right). \tag{3.28}$$

States for which the flux $\tilde{F}_-$ is non-zero, should be asymmetric with respect to the fermion number of c_+ relative to that of c_-. One non-trivial example is the following mixed state

$$|f\rangle = \prod_{i=1}^{N_f} c^{(1)\dagger}_{+i}|0\rangle = \prod_{i=1}^{N_f}\left(\frac{a^{(1)\dagger}_i + b^{(2)\dagger}_i}{\sqrt{2}}\right)|0\rangle. \tag{3.29}$$

Such a state has non-zero expectation value[30] $\tilde{F}_- = \langle J^1_R\rangle = f$. Notice that it is a $g_R = (-1)^{F_L}$ invariant mixed state of fermions and holes following the binomial distribution. If we take the large f limit, this mixed state can be well approximated by a state with equal fermion and hole numbers $f/2$. In this case we obtain the non-zero expectation values $F_+ = \tilde{F}_- = f$ with all other fluxes set to zero as we wanted.

In conclusion we can regard the thin 'tail' in the right part of figure 3 as the mixed state (3.29). Thus this Fermi sea represents a background with flux $F_+ = \tilde{F}_- = f > 0$. This statement is clear in the large f limit (weak coupling limit), where the quantum mixing can be treated classically. This mixing issue seems to be consistent with the fact that we have D-instanton corrections near the top of potential.

Finally, we would like to mention that a similar SU(2) symmetry arises even non-perturbatively. To see this let us consider the exact creation operator of fermions for each energy level (classified in terms of even and odd wave functions) in the quantum mechanics.[31] In this formulation we still have the symmetry mixing fermions and holes as in our previous discussion. We leave the spacetime interpretation of this symmetry as an interesting future problem to consider.

[30] The expectation value of J^0 in this state is zero consistent with the fact that the configuration in figure 3 describes a state with $\mu = 0$.

[31] Note that the exact potential is a double well parity invariant potential.

4. Type 0B spacetime gravity

Let us consider in more details the space-time interpretation of the 0B string backgrounds discussed in the previous section. It is instructive to start with the effective action of 0B string theory[32] [9] (written in units $2\kappa^2 = 1$):

$$S_{2d} = \int d^2x\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2 + c - (\nabla T)^2 + \frac{2}{\alpha'}T^2\right) - \frac{1}{2}e^{-2T}(\nabla C)^2 + \cdots\right]. \quad (4.1)$$

As we discussed in the previous section, when the tachyon field is set to zero (that is $\mu = 0$ in the Fermi sea picture) the system is characterized by a larger symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ asympt otically, which acts on the RR fields in 0B theory. Hence, in this section we focus on backgrounds with $T = 0$, and begin by considering the backgrounds which involve only the usual RR field, $F_\pm = \langle J^3_{L,R}\rangle$ in the notation of the previous section. It is easy to verify that the condition $T = 0$ is consistent with the tachyon equation of motion as long as a background has the property

$$(\nabla C)^2 = 0\,. \quad (4.2)$$

When this is the case, we can consistently set $T = 0$ and write the action (4.1) in the familiar form (CGHS model) [46]:

$$S_{2d} = \int d^2x\sqrt{-g}\left[e^{-2\phi}(R + 4(\nabla\phi)^2 + 4\lambda^2) - \frac{1}{2}(\nabla C)^2\right], \quad (4.3)$$

where we introduced $\lambda = \frac{\sqrt{c}}{2}$, cf. [46]. In the light-cone variables $x^\pm = x^0 \pm x^1$, the field strengths are

$$F_+ = \partial_+ C\,, \qquad F_- = \partial_- C\,. \quad (4.4)$$

Since for the backgrounds with zero tachyon field, $T = 0$, the effective dynamics of 0B theory reduces to that of the CGHS model, we can use the results of [46] to write down the most general solution consistent with (4.2). Specifically, in the conformal gauge

$$\begin{aligned} g_{+-} &= -\frac{1}{2}e^{2\rho}\,, \\ g_{++} &= g_{--} = 0\,, \end{aligned} \quad (4.5)$$

the equations of motion look like:

$$\begin{aligned} e^{-2\phi}(4\partial_+\rho\partial_+\phi - 2\partial_+^2\phi) + \frac{1}{2}F_+^2 &= 0\,, \\ e^{-2\phi}(4\partial_-\rho\partial_-\phi - 2\partial_-^2\phi) + \frac{1}{2}F_-^2 &= 0\,, \\ e^{-2\phi}(2\partial_+\partial_-\phi - 4\partial_+\phi\partial_-\varphi - \lambda^2 e^{2\rho} &= 0\,, \\ -4\partial_+\partial_-\phi + 4\partial_+\phi\partial_-\phi + 2\partial_+\partial_-\rho + \lambda^2 e^{2\rho} &= 0\,, \\ \partial_+\partial_- C &= 0\,. \end{aligned} \quad (4.6)$$

[32]Notice, that the normalization of the R-R field C here differs from the standard normalization in 0B theory: $C \to C/\sqrt{4\pi}$. This choice of conventions is justified by connection with two-dimensional gravity models [46] discussed below.

The most general solution to the equations (4.6) can be written in terms of two free fields, called u and w in [46]. Furthermore, the general solution to the C-field equation of motion consists of two plane waves: one left-moving and one right-moving. In order to meet the consistency condition (4.2), we have to allow only one type of modes. This corresponds to setting $F_+ = 0$ or $F_- = 0$. Without loss of generality, we can assume that $F_- = 0$, i.e.

$$C = C(x^+) \,. \tag{4.7}$$

Then, the most general solution to (4.6) can be expressed in terms of a single function, $u = u(x^+)$, which is related to the light-like matter field C via

$$F_+^2 \equiv (\partial_+ C)^2 = -2u'' \,. \tag{4.8}$$

The function $u(x^+)$ plays the role of the matter stress-energy tensor:

$$u(x^+) = \frac{M_0}{\lambda} - \int dx^+ \int dx^+ T_{++}(x^+) = \frac{M_0}{\lambda} - \frac{1}{2}\int dx^+ \int dx^+ F_+^2 \,. \tag{4.9}$$

The function $u(x^+)$ also determines the metric (4.5) and the dilaton

$$e^{2\phi} = e^{2\rho} = \frac{1}{u(x^+) - \lambda^2 x^+ x^-} \,. \tag{4.10}$$

This is the most general solution (written in the $w = 0$ gauge) that depends only on x^+. Since such solutions satisfy the condition (4.2), they all represent consistent backgrounds in 0B string theory.

Sometimes, it is convenient to write the general solution (4.10) in a different set of coordinates, so that one of the coordinates is precisely the dilaton, ϕ. Since both ϕ and ρ in (4.10) have simple and universal dependence on x^-, we replace x^- by ϕ. (Since the solution has interesting dependence on the second light-cone variable, x^+, it makes sense to leave this variable intact). Hence, we need to rewrite (4.10) in terms of the variables (ϕ, x^+). A straightforward calculation gives the metric in the linear dilaton variables:

$$ds^2 = \frac{2}{\lambda^2 x^+} d\phi dx^+ - \frac{1 + (x^+ u' - u)e^{2\phi}}{\lambda^2 x^+}(dx^+)^2. \tag{4.11}$$

Now, let us consider a simple solution that corresponds to a background with the light-like R-R flux given by $F_+ = -f/\sqrt{x^+}$. Substituting this flux value into (4.9), we find

$$e^{2\phi} = e^{2\rho} = -\frac{1}{f^2 x^+ \log x^+ + \lambda^2 x^+ x^-} \,, \tag{4.12}$$

where, for simplicity, we set $M_0 = 0$. Let us define a new set of coordinates

$$x^+ = e^{-y^+} \,, \qquad f^2 \log x^+ + \lambda^2 x^- = -e^{-y^-} \,, \tag{4.13}$$

where $y^+ = \phi + t$, $y^- = \phi - t$. Then the background (4.12) can be written as

$$ds^2 = \frac{f^2}{\lambda^2} e^{y^-} (dy^+)^2 + \frac{1}{\lambda^2} dy^+ dy^- \,, \tag{4.14}$$

and the dilaton is given by ϕ. In these coordinates, the field strength is time-dependent, $F = f e^{-y^+/2} dy^+$.

It is easy to see that this background describes a space-time, where particles can freely propagate into the strongly coupled region. Using (3.6), we can interpret it as a time dependent Fermi sea (see [69, 25] for similar discussions in the two dimensional bosonic string theory) (see figure 5)

$$\Big\{(x,p) \mid (p+x)(p-x-fe^{-t/2})<0\Big\} . \tag{4.15}$$

Another important example of a flux background in 0B gravity corresponds to the Fermi sea (3.14). This background describes a static space-time with two types of fluxes turned on:

$$F_+ = f\,, \qquad \tilde{F}_- = f\,. \tag{4.16}$$

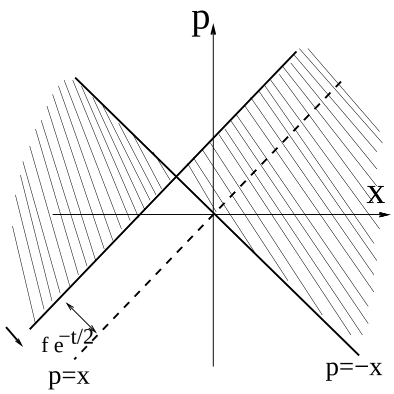

Figure 5: The time-dependent Fermi sea for the 0B flux background with flux $F_{y^+} = fe^{-y^+/2}$.

Notice, that the tachyon tadpole is zero for this background since the fields F and $\tilde{F}$ couple differently to the tachyon, so that their contributions cancel. A simple way to see this is to note that the background (4.16) is invariant under the $\mathbb{Z}_2$ symmetry (3.15). Hence, one can consistently set $T=0$. The stress-energy tensor for the fields (4.16) is given by

$$T_{++} = \frac{1}{2}F_+^2 = \frac{f^2}{2}\,, \qquad T_{--} = \frac{1}{2}\tilde{F}_-^2 = \frac{f^2}{2}\,. \tag{4.17}$$

Therefore, this system is again formally equivalent to the CGHS system with non-zero flux $F_\phi = f$. The corresponding solution is given by

$$e^{2\phi} = e^{2\rho} = -\frac{1}{a - f^2\log|x^+x^-| + x^+x^-}\,. \tag{4.18}$$

After a coordinate change

$$x^+ = -e^{-y^+}\,, \qquad x^- = e^{-y^-}\,, \tag{4.19}$$

we obtain the metric

$$ds^2 = \frac{dy^+dy^-}{1-(a+2f^2\phi)e^{2\phi}}\,, \tag{4.20}$$

and the string coupling

$$g_s^2 = \frac{e^{2\phi}}{1-(a+2f^2\phi)e^{2\phi}}\,. \tag{4.21}$$

Notice, that if we set $f = iq/\sqrt{8\pi}$ and $a=m$, we obtain the T-dual (3.1)–(3.2) of the extremal 0A black hole.

Now let us examine the tachyon equation of motion in this spacetime. Using a simple field redefinition, it is easy to show that the low-frequency tachyon modes obey a Shrödinger-like equation (2.42) with the effective potential, $V_{\rm eff}$, which in the asymptotic region looks like

$$V_{\rm eff} \simeq -(a+2f^2+2f^2\phi)e^{2\phi}\,. \tag{4.22}$$

As expected from the matrix model, tachyons are repelled from the strongly coupled region. Finally, we point out that the f-dependence of the time delay can be estimated as in 0A case, and leads to the result, f^{-iP}, consistent with the matrix model expectation.

5. Type-IIB string in two dimensions and the matrix model dual

As we have seen the Fermi sea of figure 3 is invariant under the $\mathbb{Z}_2$ transformation (3.15), which can be regarded as particle-hole duality. Thus we can orbifold the 0B theory with RR-flux by this operation. Since this is equivalent to imposing a GSO projection, we expect that the projected matrix model describes type-IIB theory with RR-flux. In the end we find a single Fermi surface as shown in figure 6.

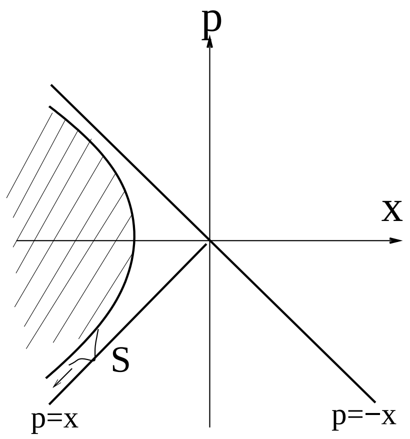

Figure 6: Fermi sea for type-II string.

Thus we conclude that the projected Fermi sea describes type-II string in two dimensions. The field content of this theory[33] is given by a left-moving RR scalar field $C(x^+)$ and a right-moving fermion ψ_{RNS}. The latter can be bosonized[34]

$$\psi_{RNS}(x^-) = e^{i\hat{C}(x^-)/\sqrt{2}}. \tag{5.1}$$

Note that the way we get $\hat{C}(x^-)$ here is quite similar to the one to obtain $\tilde{C}(x^-)$ from $C(x^-)$ in type 0B case (see (3.16)).

Now we get one regular scalar field $C(x^+) + \hat{C}(x^-)$ and this is consistent with the collective field theory of the previous Fermi sea picture. From the Fermi sea picture we can see that the perturbative scattering amplitudes of $C(x^+)$ and $\hat{C}(x^-)$ are the same as tachyon scattering amplitudes in the usual $c = 1$ matrix model. The string coupling is proportional to the inverse of the RR-flux $F_+ = \hat{F}_- = f$; thus the flux regulates the strongly coupled region.[35] Notice also that in this background the non-perturbative instability still exists as in 0B case since the $\mathbb{Z}_2$ identification does not eliminate tunneling effects. This is consistent with the fact that type IIB still has D-instantons. This instability is well suppressed when f is large.[36]

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[33] We can impose the locality condition (GSO projection) as follows (see e.g. [8]). The chiral OPE gives (ϵ is the chirality in 2D) $V_{NS}(z)V_{NS}(0) \sim z^{-1}$, $V_{NS}(z)V_{R(\epsilon)}(0) \sim z^{-\frac{1}{2}}$ and $V_{R(\epsilon)}(z)V_{R(\epsilon')}(0) \sim z^{\frac{\epsilon\epsilon'-1}{4}}$, where we have omitted the part which depends on ϕ and X^0. Thus to maintain the locality we should choose $\{(R(-), NS), (NS, R(-)), (R(+), R(+))\}$ in type IIB.

[34] This implies that the RR scalar (axion) has the compactification radius of a free fermion. This is different from that of the axion in type 0B string theory (self-radius). However, this is consistent with the fact that the RR charge of a D-brane also differs by factor $\sqrt{2}$.

[35] See [20] for discussions on two dimensional type-IIB string theory in $N = 2$ Liouville theory and its matrix model dual. Obviously their background considered is different from ours.

[36] Even though the Fermi sea looks the same as that in bosonic string, there is an important difference. After we include tunneling effects, the original Fermi sea disappears in the bosonic string case, while the Fermi sea will be quantum mechanically mixed in type-IIB case.

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